Problem Set #1

PHY 300 Observational Astronomy

Throop / Booth

ASSIGNED: MONDAY 9-FEB-2015
DUE: MONDAY 16-FEB-2015, BEGINNING OF CLASS

1. [30 points] **Magnitudes**. A star cluster has 100 stars in it. Each star has identical brightness of magnitude 18.

   a) [15] What is the cluster's total integrated magnitude ('apparent magnitude')?

   By the definition of magnitude, each increase of 2.5x in flux signifies one magnitude. Similarly, 5 magnitudes corresponds to a 100x increase in flux (since $2.5^5 \sim 100$). Since the cluster has 100x the flux of a single star (V=18), its total magnitude is V=13.

   Or mathematically:

   F_1 = flux of one star

   F_2 = flux of 100 stars = 100 F_1

   $\Delta m = -2.5 \log_{10}(F_1/F_2) = -2.5 \log_{10}(1/100) = 5$

   b) [15] If the cluster were moved to ten times its original distance, what would happen to its angular diameter? To the magnitude of each star? To its total apparent magnitude?

   As the cluster gets further away, its angular size decreases proportionally. Its angular diameter is 0.1x what it was before. The total flux received at Earth is 100x less ($1/r^2$ radiation). If the whole cluster had a magnitude V=13 before, it now has a magnitude V=18. The magnitude of each individual star similarly decreases, from V=18 to V=23.

   NB: The important thing to note here is that magnitude (aka 'apparent magnitude') definitely changes as a function of distance. It is not an intrinsic property of the object. Astronomers sometimes use 'absolute magnitude,' which is defined as the brightness if something is see from 10 pc — this quantity is independent of distance, but that's not what we're talking about here.

2. [30 points] **Resolving Power**. In class, we showed that NASA's Hubble Space Telescope (HST; diameter = 2.4 meter; wavelength = 0.5 um) can resolve Pluto (assume 30 AU from Earth, and diameter 2400 km) to roughly two pixels — that is, HST's resolving power is slightly smaller than Pluto's angular diameter. In order to explore Pluto better, in 2006 NASA sent the New Horizons spacecraft toward Pluto, where it will arrive later in 2015. New Horizons has a small telescope (D = 21 cm), but it will get very close to Pluto (10,000 km).

a) [15] At what distance from Pluto will New Horizons' resolution exceed that of HST?

The angular resolution of a telescope is $\theta = 1.22\ \lambda/D$, so the smallest feature it can resolve is just $r = \theta\ R = 1.22\ \lambda\ R\ /\ D$. We want to find at what distance R it is true that $r_{HST} = r_{NH}$.

$r = 1.22\ \lambda\ R_{HST}\ /\ D_{HST} = 1.22\ \lambda\ R_{NH}\ /\ D_{NH}$.

$R_{HST}\ /\ D_{HST} = R_{NH}\ /\ D_{NH}$.

$R_{NH} = R_{HST}\ D_{NH}/D_{HST}$ = (30 AU) (21 cm) / (2.4 meter) = 2.6 AU.

Interesting: Pluto is 33 AU away, but Hubble is so good that it outperforms New Horizons until NH is almost all the way to Pluto.

b) [15] New Horizons will fly past Pluto at 10,000 km above the surface. Assuming perfect optics, what are the smallest features it will be able to resolve?

$r_{NH} = 1.22\ \lambda\ R_{NH}\ /\ D_{NH}$ = 1.22 (500 nm) (10,000 km) / (21 cm) = 29 meters

3. [30 points] **More Magnitudes.** In class, I said that if you took Pluto and the full Moon and placed them each at 1 AU, they would be of similar brightness. That's not quite true, because in reality their sizes and albedos are not quite identical.

a) [15] If you were standing on the Sun, what would Pluto's magnitude be if it was at 1 AU from the Sun?

Pluto will appear brighter to you for two reasons. First, it will be closer to the Sun, and thus receiving more solar flux on its surface. Second, it will be closer to the observer, who thus gets more of Pluto's flux. Each of these relationships follows a $1/r^2$ law.

F_0 = current flux received at Earth from Pluto

F_1 = flux received at Sun from a Pluto at 1 AU

$F_1 = F_0 * (33\ \text{AU} / 1\ \text{AU})^2 * (33\ \text{AU} / 1\ \text{AU})^2 = 33^4\ F_0 = 1.2 \times 10^6\ F_0$.

Note that the flux varies as R^{-4} : perhaps surprising, but true here, just like it would be for the flux received from a mozzie in your headlights.

How many magnitudes Δm is this?

$\Delta m = -2.5 \log_{10} (F_1/F_0) = -2.5 \log_{10} (1.2 \times 10^6) = -15.9$.

→ Pluto will get 15.9 magnitudes brighter. This moves it from V=14, to V = -1.9. That's slightly brighter than the brightest star.

b) [15] What about the Moon?

Here, we keep the Moon at its present distance from the Sun (1 AU), but move the observer from 400,000 km away, to 1 AU away. (Since the Sun-Moon distance is fixed here, the flux varies just as R^{-2}, not R^{-4}.) Using the inverse square relationship,

F_0 = Lunar flux received at Earth, nominal

F_1 = Lunar flux received at the Sun, with Moon at 1 AU

$F_1 = F_0 \, (400{,}000 \text{ km} / 1 \text{ AU})^2 = 7.1 \times 10^6 \, F_0$.

How many magnitudes is this?

$\Delta m = -2.5 \log_{10} (F1/F1) = -2.5 \log_{10} (7.1 \times 10^{-6}) = 12.9$.

→ The moon will get 12.9 magnitudes fainter. Taking its magnitude to be V = -13, it will go to V=0.1.

Taken together, do these make sense? Seen from the Sun, the Moon will be about 1.8 magnitudes fainter than Pluto at 1 AU. That seems reasonable: Pluto's albedo is about 10x higher than the Moon's (a = 0.5 vs. 0.05). The moon's diameter is slightly larger than Pluto's (1700 km), so its surface area is about 2.5x that of Pluto. But the albedo difference is larger than this.

You can assume that Pluto's current distance is 33 AU from both the Sun and Earth, and it has a magnitude V = 14 (from Earth or Sun)

For the Moon, assume a distance 400,000 km (from Earth), distance 1 AU (from Sun), V = -13 (seen from Earth).

4. [10 points] **Thermal Balance**

a) [5] What would Pluto's temperature be if placed at 1 AU? Assume an albedo 0.5.

$$T_E = \left[\frac{(1-a)L_\odot}{16\,\pi R^2} \frac{1}{\sigma} \right]^{1/4}$$

$a = 0.5$; $L_\odot = 4 \times 10^{33}$ erg/sec; $\sigma = 5.67 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}; R = 1 AU

→ T_E = 236 K.

Does this seem reasonable? It's a bit of a crazy idea to move Pluto to 1 AU, but if you did it, the temperature is roughly Earth's temperature, which is what you expect in energy equilibrium. Pluto is more reflective than Earth (a = 0.5 for Pluto; 0.4 for Earth) so it would be a bit cooler than Earth. Also, the equation for T_E ignores entirely the effect of the atmosphere (the 'greenhouse effect') which in essence

provides another energy input term, because it traps in infrared energy that can't radiate to space and is blocked.

If you did this for Pluto, keep in mind that things would change fast: Pluto is mostly made of very volatile ices like CO and N_2, which would quickly sublimate, so you'd have a very short-lived planet!

b) [5] What about the Moon (albedo 0.05)?

a = 0.05

$\rightarrow T_E = 277$ K.

Does this seem reasonable? The Moon's lower albedo means a slightly higher equilibrium temperature compared with Earth (or a 1-AU Pluto). Sounds OK. Note that this is an 'average' temperature — daytime will be higher, night-time lower — and the astronauts still had to wear plenty of insulation because the lunar atmosphere is so thin that despite its moderate temperature, it doesn't actually warm the body very efficiently.