The Orbits of Jupiter's Galilean Satellites 29 May 2015

Summary

Jupiter and its four Galilean satellites (Io, Europa, Ganymede and Callisto) were observed over a period of 14 days, from 5 May to 19 May (2015). The distance of each moon from Jupiter was then plotted against time. From this the orbital period of each moon could be determined. To verify whether the orbital periods obtained were correct, they were used to determine an inferred mass of Jupiter (which has an accepted value of $1.9x10^{27}$ kg). The results obtained are summarized in the table below.

Table 1: Main results of the observations

	Io	Europa	Ganymede	Callisto
Orbital Distance (pixels)	73.71	117.3	187.1	329.1
Orbital Distance (km)	421742	670826	1070505	1882875
Orbital Period (days)	1.77	3.54	7.15	16.9
Mass of Jupiter (kg)	$1.897x10^{27}$	$1.909x10^{27}$	$1.905x10^{27}$	$1.859x10^{27}$

Using this information, each of the four satellites can be identified in each of the taken images. An example is shown below.

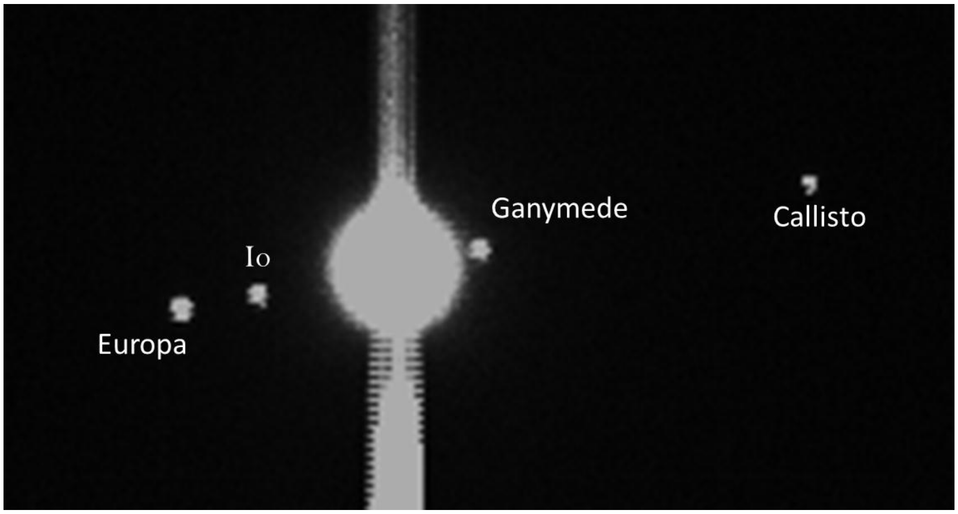

Figure 1: An image taken of Jupiter on 5 May 2015 at 21h19, with each of the moons identified

Equations

The following equations were used in this practical. The resolution of an image is given by:

$$\theta = \frac{r}{R} \quad (1)$$

Where θ is the number of pixels, r is the radial distance and R is the distance to Jupiter. The image scale is defined as one radian over the focal length.

$$s = 1/f \quad (2)$$

The distance of any moon from Jupiter can be modeled using a sinusoidal wave:

$$d = r sin(\frac{2\pi t}{p} + \beta) \quad (3)$$

Here, d is the distance between two objects (in this case Jupiter and its moon); p is the orbital period (hours is used for this practical) and β is some phase constant. To verify that the obtained orbital periods are correct, the inferred distance to Jupiter can be calculated using:

$$M = \frac{4\pi^2 r^3}{p^2 G} \quad (4)$$

Method

Each image was analyzed using the image viewer SAOImage DS9. The central position (in pixels) of Jupiter and each of its moons was determined. This was then used to determine the distance of each moon from Jupiter (again in pixels). Using equation (2), the size of each pixel could be determined; and was found to be 8.5 µm; which confirms the given value. Next the size of each pixel in kilometers was calculated using equation (1). It was found that each pixel represents 5722 km. The distance of each moon from Jupiter was then plotted against time. The following figure was obtained.

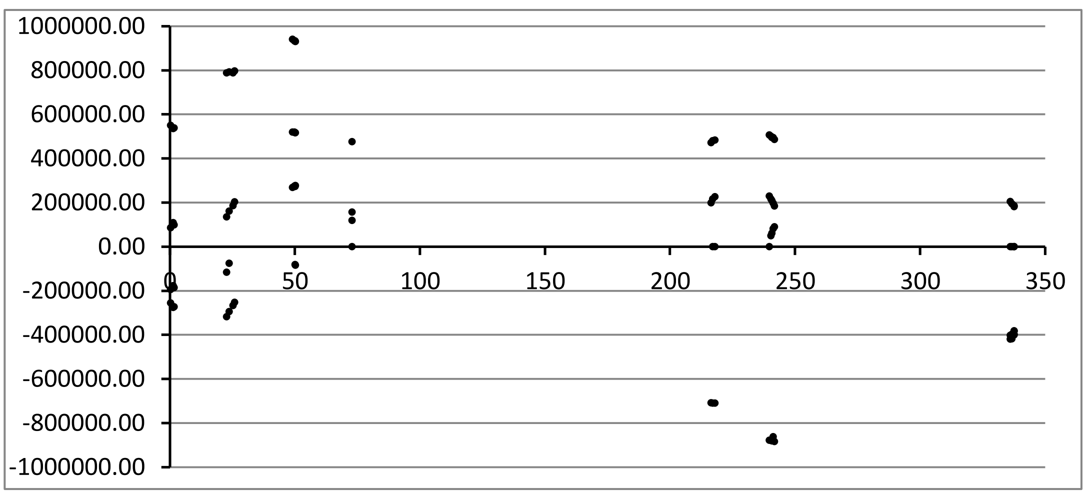

Figure 2: Unfitted and unadjusted distances (y-axis) plotted against time (x-axis)

From here, four sine curves were to be fitted to the data. This was done largely by trial and error. The following ideas were used during the process:

1. Callisto should have the largest radial orbit. Any outlier points should therefore be fitted to the curve representing Callisto's orbit. The outliers were determined to be those around the 50 hour mark and those around the 240 hour mark.

2. The images were analyzed to determine which of the moons was the brightest. This was done by analyzing the DN for each moon's center. From this, all the brightest moons were noted as possibly being Ganymede. The fact that not all of them represented Ganymede was kept in mind, as a moon could appear brighter depending on its position. This, combined with the knowledge that Ganymede has the second largest radial orbit, allowed for a plot representing Ganymede to be made. This was made easier by knowing that any points representing Callisto had most likely been eliminated, thus the "new" outliers should be Ganymede.
3. At this point, all that was left to plot was Europa and Io's orbitals. Here, the starting point was taken to be the data points in the region between the 200 hour and 250 hour marks. The points between Ganymede and the axis seemed as if they belonged on the same curve, so one was fitted to go through them. The fitting was done by alternating the approximate orbital distances for Io and Europa. The curve was found to fit Europa the best, taking into account the other data points.
4. The last step was to fit the orbital of Io. Most of the data points which were not yet fitted found a place on the fit for Io.
5. The last step was to account for any points which had not yet been fitted. It turned out that all of these could be corrected by simply changing the sign of the distance.

Results

At this point, all the data points lay on one for the four curves. The orbital period could be deduced from this, using equation (3).

Table 2: The orbital period of each of Jupiter's Galilean moons (in hours)

Io	Europa	Ganymede	Callisto
42.5	85	171.5	405

From here, the orbital distance in pixels could be determined using equation (3). These could then be converted to kilometers using the result calculated earlier that each pixel represents 5722 km.

Table 3: Orbital distance of Jupiter's Galilean satellites in pixels and in kilometers

	Io	Europa	Ganymede	Callisto
Distance (pixels)	73.71	117.3	187.1	329.1
Distance (km)	421742	670826	1070505	1882875

To check the validity of each orbital period, the mass of Jupiter was calculated using equation (4), and then compared to the accepted value.

Table 4: Inferred mass of Jupiter (kg) due to the orbital period of each moon

Io	Europa	Ganymede	Callisto
1.897×10^{27}	1.909×10^{27}	1.905×10^{27}	1.859×10^{27}

The fits for the orbital periods, showing how each curve goes through the data points, is displayed in figure 3.

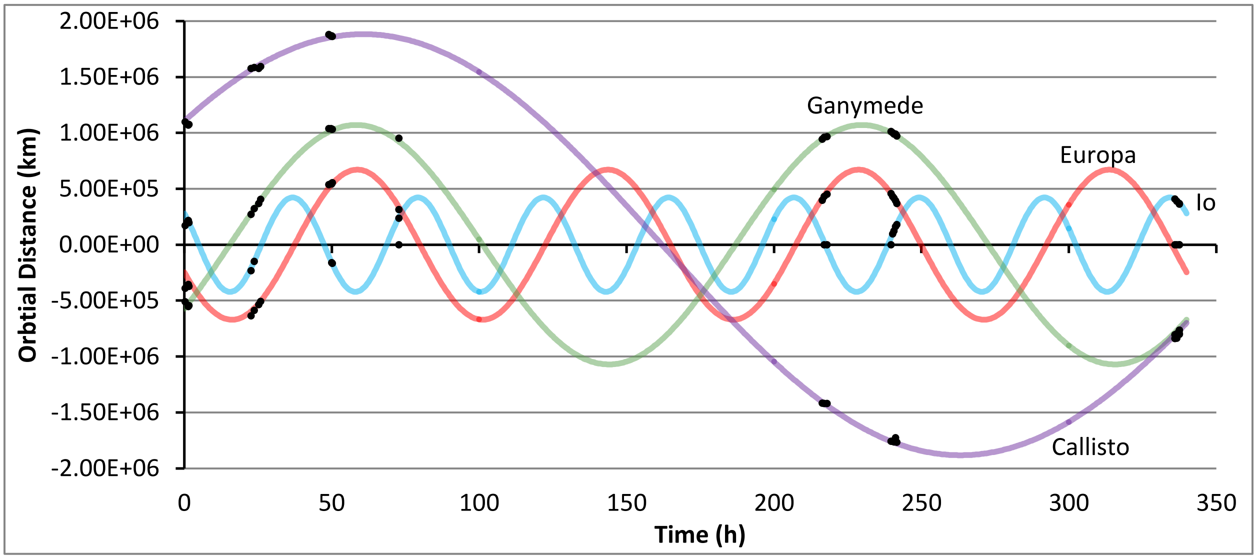

Figure 3: Fitted orbital periods for each of the four Galilean moons of Jupiter