Problem Set #5

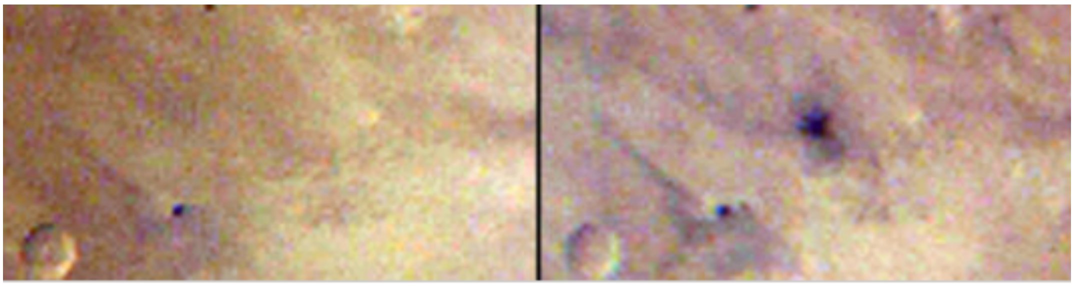
PHY 300 Observational Astronomy

Throop / Booth

ASSIGNED: WEDNESDAY 22-APR-2015
DUE: WEDNESDAY 6-MAY-2015, BEGINNING OF CLASS

1. Martian Crater Mining [30 points]

You are searching through data from a Mars mission. You find a pair of images, seen below. In it you see where a new crater has just been made on Mars. You measure it to be 50 meters in diameter.



The left image was taken during Martian afternoon on March 27, 2012; the right one on the afternoon of March 28, 2012 by the Mars Color Imager (MARCI) weather camera on NASA's Mars Reconnaissance Orbiter.

a. What is the impactor velocity? How much energy (in kJ) is dissipated in this making this crater? You can assume that the impactor comes in to Mars from infinity. You can use the following assumptions:

- (Crater depth)/(crater diameter) = 0.1
- (Impactor diameter)/(crater diameter) = 0.1
- Impactor has a density equal to that of Mars (ρ = 4 g/cm^3)
- Ejecta has a uniform velocity of 1 km/sec.

Impactor velocity. We assume that the velocity is zero at infinity. Thus, we can calculate the impactor velocity:

```
G M m / r = 1/2 mv^2
v = v_esc_Mars = sqrt(2 G M_mars / r_mars)
  = 5 km/sec.
```

Note that there are two escape velocities here. The first is that from Mars, which is above. But in order for a body to truly get to infinity, it has to also escape the Sun's gravity. We didn't talk about this explicitly, so while some people may have included this, it's OK for this PS to use either one. However, they are quite different! Here is the escape velocity away from the Sun's potential well:

```
v = v_esc_Sun = sqrt(2 G M_Sun / (1.5*AU))
  = 34 km/sec.
```

**** That is, the escape velocity from the Sun potential is many times more than that from Mars itself.**

In reality, most impactors onto Mars come from the asteroid belt, which has an orbital distance not much larger than Mars' itself, so escape velocity from the Sun is not that important. (i.e. when I said 'the impactor comes in from infinity', I really should have said 'the impactor comes in from 1.5 AU, but outside of Mars' potential')

Energy to excavate the crater is just $E_{exc} = 1/2\ m_{ej}\ v_{ej}^2$, where m_{ej} is the ejecta mass, and v_{ej} is the ejecta velocity. We can calculate the former, and are given the latter.

m_{ej} = 1/3 * pi r^2 h rho
E_{exc} = 1/2 mv^2 = 1/2d * m_{ej} * $(1*km)^2$
= 1/2 * 1/3 * pi * $(25*meter)^2$ * 5*meter * 4 * $(1*km)^2$
= 6.5e12 J

For a reality check, let's see what the incoming impactor energy is:

PE = GMm/r = G (4/3 pi r^3 rho) / r_{Mars}
= 3.3e12 J

So, our simple approximations about crater size are clearly off by a little bit, as the excavation energy exceeds slightly the incoming kinetic energy. But we're off by a factor of 2, which considering the simplifications I gave you about crater size, is not too bad.

b. About how many tons of dynamite (TNT) equivalent is this? 1 ton of TNT gives off roughly 4 x 10^9 J.

E_{exc} / (4e9) = 1636 tons of TNT. This is much greater than the mass of the impactor itself!

c. Roughly how often do you expect such a crater to be formed on Mars? Assume that Mars gets roughly double the impactor flux that we have on Earth (due to its closer proximity to the asteroid belt), and use the chart for impactor frequency presented in class

This is a 5-meter diameter impactor. Looking up in chart from Lecture 21, it's roughly every 5 years on Earth, so ~ every 2.5 years on Mars.

d. On the field trip, Danie mentioned that heavy metals in South Africa are typically found at the bottom of the Transvaal supergroup, to a depth of roughly 3 km. You decide on a brilliant new way to mine for gold: simply wait for an impact crater to excavate down to this depth, and then sweep up the debris. How large an impactor would it take to excavate a crater to this depth? How long would you have to wait for such an impactor?

We want a crater depth of 3 km, so a crater diameter of 30 km. This means an impactor size of 3 km. Looking up on the impactor frequency plot, we'd be waiting for some 5 million years to have such a crater hit anywhere on Earth.

If we want to make sure this crater hits your region of South Africa, we'd have to wait quite a bit longer. You can estimate how many times longer by taking the ratio of the crater area to the Earth's surface area: (4 * pi * r_earth^2) / (pi * (30 * km)^2) =

180,000 x longer, or roughly 1 trillion years – longer than the age of the universe. So, don't wait!

(How does it make sense that we'd have to wait such an insanely long amount of time, even though we see evidence of large craters like this all over the Moon? The impactor flux on all planets has dropped by many orders of magnitudes over time: it was much higher in the past, and is low enough now so that most places on Earth will never get hit by a 3-km body, fortunately for us.)

And to be clear, this is a crazy idea: no one would ever try to mine by using an asteroid impact. However, there is evidence that some gold deposits near Joburg were brought closer to the surface due to the Vredefort impactor.

2. Precise Exoplanet Photometry [30 points]

You are observing stars with a photometer on your 1-meter telescope. You are watching the star gamma Ori, which you know from previous work gives you about 1000 photons/second at your focal plane.

You are monitoring your star once per second. You notice that the star starts to dip in brightness, to 0.9 of its original brightness.

a. What is your SNR for the transit itself? Assume that all of your noise is from Poisson statistics — that is, you can remove any noise from dark current, flat-fielding, sky brightness, etc. very accurately.

The transit itself is a signal of 100 photons/sec (that is, the difference between the measurement before and the measurement after). Since we monitor once per second, that's 100 photons per observation. N = 100, so sqrt(N) = 10.

SNR = Signal / Noise = 100 / sqrt(900) ~ 3.

Many people got this wrong. The signal that we're looking for is the difference in brightness between before and after – that is, what we really care about measuring is the 100, not the 1000 or 900. The signal – the real science – is just 100 photons/second.

For this problem, you can use sqrt(900) or sqrt(1000) – either one makes sense physically, and it doesn't change the result much.

b. You'd like to adjust your integration time (exposure time), so that you can measure its light curve very accurately. What integration time should you choose to get an SNR of 20 during the occultation?

SNR = Signal / Noise
20 = 100*t / sqrt(900*t)
t = 36 seconds

c. The star that you are monitoring continues to be interesting, so you acquire a larger telescope and detector. This telescope is a 2-meter diameter. This new CCD has a dark-current of 10 e- sec, a bias of 200 e-, a QE of 0.5, and a gain of 10 e- per DN. What is the count rate in your detector in DN per second?

Total DN count rate = (photons/second) * QE / gain + dark-current + bias.

The telescope is 4x larger than before, so for a 1-second exposure, we have DN = (4000 photons/sec) * 0.5 / 10 + 10 + 200 = 410 DN/sec.

Note that the gain is used in calculating the signal from photons only. The dark-current and bias are already in electrons, so these are independent of the gain.

d. You would like to calculate the SNR to the same level of 20 with this new telescope. How does your exposure time here compare to that calculated in part b. (i.e., shorter or longer, and by how much?) Consider only Poisson noise.

Since the telescope has 4x the collecting area, it will get 4x the number of photons per second as before. With 4x the photons, we will

get to the same SNR in half the time, because SNR ~ sqrt(t) for Poisson statistics.

e. After much work, you measure the transit depth to be 9%, and have a period of 50 days. You know (from the star's spectrum) that it has a mass of 3 M_{sol} and a radius of 0.7 r_{sol}. What is the planet's orbital distance? Its radius? What measurements would you need to determine its mass?

Orbital distance: Using Kepler's laws, we calculate the relationship between the period and the orbital distance:

$$p^2 = (2\,pi)^2\, a^3 / (G\,M)$$

Rearrange this to get

$$a = (p/(2\,pi))\, G\,M)^{1/3}$$
$$= 0.28\ AU$$

Planet's radius: We know that the planet blocks 9% of the star's flux – that is, the planet's surface area is 9% that of the planet's surface area. So,

$$r_{planet}^2 = 0.09\,(0.7\,r_{sun})^2$$
$$r_{planet} = 0.21\,r_{sun}.$$

That's a big planet! Something like this is called a brown dwarf – larger than a planet, smaller than a star.

These measurements here are sensitive to the star's mass (via orbital period), but not the planet's mass. For the latter, you'd have to do something else, such as measure the star's wobble –– i.e., its doppler shift due to radial velocity.

3. Photometric Bands [40 points]

Astronomers typically use filters to do stellar photometry. Here, you will calculate the ratio of energy in different bands.

a. Assume you have two stars. One has a temperature of 3000 K (a very cool star that will burn for a long time), and the other has a temperature of 10,000 K (which makes it a very hot star that will burn out within a few Myr).

b. The blackbody flux is given by B(λ,T). Make a plot of the flux vs. wavelength for each star. Make sure to scale the plots appropriately — that is, choose a reasonable range for B and λ so that the curves can be clearly plotted.

   You can assume the stars to be perfect black bodies. B(λ,T) has units of W m^{-2} m^{-1}. k is the Boltzman constant (1.4 10^{-23} J K^{-1}), and h is Planck's constant (6.6 10^{-34} J sec).

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

c. What is each star's total integrated flux emitted in the red 'R' filter (i.e., between 550-700 nm)? Similarly, what are their fluxes in the ultraviolet ('U') filter (300-400 nm)? What is the ratio between these: F_U/F_R for each star? To be clear, you are essentially calculating the integral

$$\int_{\lambda_1}^{\lambda_2} B(\lambda, T) d\lambda$$

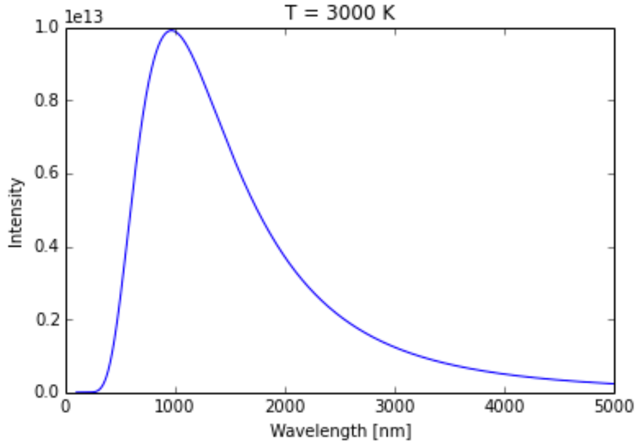
Hint: In theory you could do this problem analytically, by integrating B(λ,T). But that is quite difficult for an equation of this form. Instead, I suggest doing it numerically. You might consider approaching it this way:

i. Set up a matrix of values, something like the one below. You could do this in a spreadsheet (like Excel or Google Docs), or write a short program in something like python or MATLAB. Either one is fine.

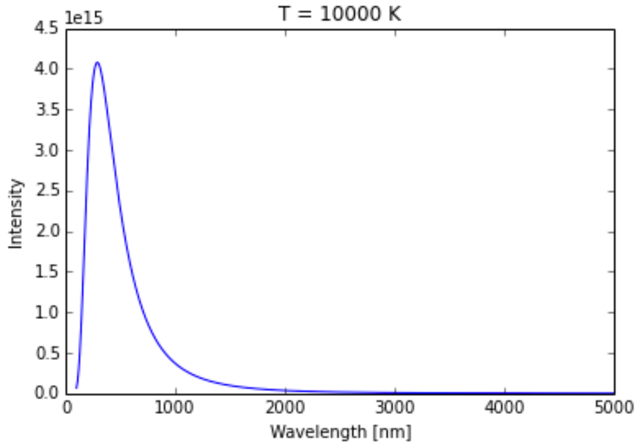
ii. Break up the spectrum into a number of different wavelength bins (perhaps 100 or 1000). For each bin, calculate both B and the flux. Note that the units of B are in flux per wavelength, so in order to get the total flux within one of your bins, you need to multiply by the bin width. (Just like you'd multiply by dλ in an analytic integral.)

iii. To calculate the flux seen through a given filter, sum the flux emitted across the wavelengths covered by that filter (e.g., the total flux from 550 to 700 nm). You are numerically integrating the blackbody equation here. This is common to do for such heinous equations which are difficult to manually integrate.

Many people used Maple for this, which is excellent. I have posted solutions in both Python and Excel. (Excel is really not the best way to solve this because it hides the equations, and it's tedious dealing with large datasets. But for those experienced with it, it does do the job.)




```
Ratio Flux Blue (300:400) / total = 0.002
Ratio Flux Red  (550:700) / total = 0.061
Ratio Flux Blue/Red               = 0.035
```




```
Ratio Flux Blue (300:400) / total = 0.214
Ratio Flux Red  (550:700) / total = 0.119
Ratio Flux Blue/Red               = 1.791
```

```python
# Program to calculate energies in specific passbands of a black body spectrum
# PHY 300 PS5, May 2015
# Henry Throop [solution set]
# Python version

import numpy as np
import matplotlib.pyplot as plt
from pylab import *

##########
def bb_lam( lam, t ):            # return blackbody intensity, energy cm^-2 sr^-1 s^-1 cm^-1
##########
    c               = 3e10          # speed of light, cm s^-2
    k_boltzman      = 1.38e-16      # boltzman const, erg k^-1
    h_planck        = 6.626e-27     # planck's constant, erg-sec

    return 2 * h_planck * c**2 / lam**5 * 1 / (exp(h_planck * c / ( lam * k_boltzman * t)) -1)

##########
def plot_spect (lam, i, t):      # Plot a spectrum
##########
    nm   = 1e-7  # nm, in cm

    plot(lam/nm, i)
    plt.xlabel('Wavelength [nm]')
    plt.ylabel('Intensity')
    plt.title('T = ' + repr(t) + ' K')

    plt.show()

##########
def calc_spect_bands (lam, i, t): # Calculate energy in the spectral bands
##########
    nm   = 1e-7  # nm, in cm

# Calculate the total integrated flux

    flux_tot = sum(i * dlam)

# Calculate the flux in R-band (red, 550-700 nm) and U-band (300-400 nm)

    flux_300_400 = sum((i * dlam)[(lam > 300*nm) & (lam < 400*nm)])
    flux_550_700 = sum((i * dlam)[(lam > 550*nm) & (lam < 700*nm)])

    print "Ratio Flux Blue (300:400) / total = " + repr(round(flux_300_400 / flux_tot, 3))
    print "Ratio Flux Red  (550:700) / total = " + repr(round(flux_550_700 / flux_tot, 3))
    print "Ratio Flux Blue/Red               = " + repr(round(flux_300_400 / flux_550_700,3))

##########
## MAIN PROGRAM
##########

# Initialize constants and units

nm   = 1e-7              # nm, in cm
um   = 1e-4              # microns, in cm

# User-controlled input variables

lam0 = 100*nm            # Set lower range of wavelengths in spectrum
lam1 = 5*um              # Set upper range of wavelengths in spectrum
nlam = 1000              # Set the number of wavelength bins
temps = [3000, 10000]  # Do these stellar temperatures

# Create the wavelength array

dlam = (lam1-lam0) / nlam
lam = np.arange(lam0, lam1, dlam)

# Do the calculations for each stellar temperature

for t in temps:
  i = bb_lam(lam, t)
  plot_spect(lam, i, t)
  calc_spect_bands(lam, i, dlam)
```