Exam 1 Solutions

1. [10 points]

From the stars' spectra, you can infer their temperatures T. By Wien's law ($\lambda_{\text{max}} \sim 1/T$), the star that is red must have a cooler temperature than the star that is blue.

Hotter stars also have a much higher luminosity, which increases as $L = AF = A\sigma T^4$. The blue star must have a much higher luminosity than the red star. In order for their fluxes at Earth to be equal, the blue star must be further away than the red one.

The surface area A of a star also increases with temperature, but we haven't talked about that in detail yet. With what we've done in class it's not possible to calculate quantitatively the relative luminosities between these two stars – but we can definitely compare their distances qualitatively.

NB: There are very few stars that appear doppler-shifted to a naked-eye observer. Galaxies can be doppler-shifted – typically red-shifted, because they are moving away from us at a substantial fraction of the speed of light. But all of the stars we can easily see are within the Milky Way – our own local galaxy – and are moving at speeds of maybe 10's to 100's of km/sec, nowhere close to c.

2. [25 points: 15 / 5 / 5]

We want to take Spica's magnitude, and convert that to a flux. To do so, we can use the known relationship that the Sun has a magnitude $M_v = -26.7$, and a luminosity $L_\odot = 4 \times 10^{26}$ J/sec.

Flux from Sun:

$$F_\odot = L_\odot/(4\pi R^2) = L_\odot/(4\pi(1\text{AU})^2) = 1422\,\text{Wm}^{-2}$$

(Occasionally in class we used $F_\odot \approx 1500\,\text{W}\,\text{m}^{-2}$, and I don't mind if you used that instead of calculating it from $L_\odot$.)

So the flux from Spica is just

$$F_{sp} = 2.5^{\Delta m} F_\odot = 25^{-1-26.7} F_\odot = 2.5^{-27.7} F_\odot = 1.35 \times 10^{-8}\,\text{Wm}^{-2}.$$

b. Each photon has an energy

$$E_\gamma = h\nu = hc/\lambda = hc/(500\,\text{nm}) = 4 \times 10^{-19}\text{J},$$

so the total photon number flux is

$$n = F/E_\gamma = 3.4 \times 10^{10}\,\text{photons sec}^{-1}\,\text{m}^{-2}.$$

NB: I mistakenly wrote $\hbar$ instead of h on the equation sheet, so no deductions if your answer was off by 2π. The values above are correct.

c. The number of photons reaching your eye is just the flux times the cross-sectional area of your eye, or

$$n = FA = (3.5 \times 10^{10})(\pi(0.0035\ \mathrm{m})^2) = 1.4 \times 10^6\,\mathrm{photons\,sec^{-1}}.$$

3. [15 points]

During each oscillation, the star increases its radius by a factor of 3 (say, from r_0 to $r_1 = 3r_0$). Its temperature T stays constant, and thus so does its surface flux F. Because the luminosity $L = FA$ is a function of surface area A, the star radiates more energy at size r_1 than r_0. The relative luminosities are:

$$L_0 = \sigma T^4 A_0 = \sigma T^4 (4\pi r_0^2)$$
$$L_1 = \sigma T^4 A_1 = \sigma T^4 (4\pi (3r_0)^2)$$
$$L_1/L_0 = A_1/A_0 = (3r_0)^2/(r_0)^2 = 9.$$

So, the star's luminosity changes by a factor of 9. How many mangitudes is this?

$$\Delta m = -2.5 \log(L_1/L_0) = -2.5 \log(9/1) = -2.4$$

The star gets 2.4 magnitudes brighter during its oscillations. Sounds pretty reasonable. That's easily detectable to a human observer. Some variable stars do have magnitude swings of about this amount.

4. [15 points]

A solar day is the time (averaged throughout the year) to go from one sunrise to the next sunrise – that is, for the Earth to rotate once on its axis, relative to the Sun. It's the 'day' that we're familiar with: 24 h. There are 365 solar days per year.

A sidereal day is the time for the Earth to rotate once relative to the stars. This is a little bit shorter than a solar day, because by the time one sidereal day has passed, the Earth has moved a little bit forward in its orbit, making the Sun's position change. It requires another few minutes for the Earth to rotate once relative to the Sun. Sidereal day = 23h56.

Another way to think of it: the Sun rises and sets 365 times per year. But the stars rise and set 366 times per year – with the difference being due to the fact that the Earth's *revolution* around the Sun causes the Sun to rise and set one fewer time.

5. [25 points: 15 / 5 / 5 / 5]

From the focal ratio, we know that the focal length f is

$$f = 20\,\mathrm{m}.$$

Also from the focal ratio, we know the plate scale at the focal plane:

$$p = 1/f = 1/20\,[\mathrm{rad\ m^{-1}}] = 1/20,000\,[\mathrm{rad\,mm^{-1}}]$$
$$= 206264/20000 = 10.3\,[''\,\mathrm{mm^{-1}}],$$

where 206264 is the number of arcseconds per radian:

$$(1\,\mathrm{rad})\left(\frac{360^\circ}{2\pi\,\mathrm{rad}}\right)\left(\frac{60'}{1^\circ}\right)\left(\frac{60''}{1'}\right) \approx 206264''\,\mathrm{rad}^{-1}.$$

Angular separation of Jupiter and Ganymede is

$$\theta = r/R = 200,000\,\mathrm{km}/5\text{ AU} = 0.00027\,\mathrm{rad} = 55''.$$

So, the separation on the film is just the ratio of these, or

$$dx = \theta/\mathrm{p} = 55''/10.3'' = 5.4\,\mathrm{mm}.$$

b. The smallest feature we can see on Jupiter is specified by larger of the atmospheric seeing limit, and the telescope's diffraction limit.

Diffraction limit:

$$\theta = 1.22\lambda/D = 6 \times 10^{-7}\,\mathrm{rad} = 0.12''.$$

Atmospheric limit:

$$\theta = 1''.$$

The atmospheric limit is larger and thus the important one here – it limits the resolution to be much worse than the telescope's intrinsic diffration limit.

The smallest resolvable feature then is just

$$r = \theta R = (1'')(5\mathrm{AU}) = (1/206264)(5)(1.5 \times 10^{11}\mathrm{m}) = 3600\,\mathrm{km}.$$

c. In ideal conditions, we could see to the diffraction limit:

$$r = \theta R = (0.12'')(5\,\mathrm{AU}) = 435\text{ km}.$$

d. Older telescopes had long focal lengths in large part to remove aberrations (mostly spherical aberration, also field-flattening), which are smallest at high f/ratios. The large plate scale due to a long focal length also matched well the relatively large 'pixels' of photographic plates. New technology allows better construction of low-aberration optics, allowing for 'faster' optical systems.

(This change is unrelated to the shift from lenses to mirrors: both of these have aberrations. Mirrors allow for a shorter tube because the optics can be folded, but for a given diameter, will have the same focal ratio as a refractor would. But for the context of this question, mirrors vs. lenses is not relevant.)

6. [10 points]

If the full moon is setting, the sun must be rising. At sunrise, it must be early morning, and you are thus eating breakfast.